CIRCULATING AND JET FLOWS FORMED IN THE ATMOSPHERE DURING THE RISE OF TWO LARGE-SCALE THERMALS

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The study of the dynamics of large-scale eddies and thermals in the atmosphere is important for various applications in meteorology, ecology, and the theory of combustion and explosion. Although the laws governing the rise of individual objects have been studied with sufficient thoroughness (see [1-5], for example), there has been much less attention given to the laws governing the rise and interaction of paired objects [6-10]. In the present study, we numerically investigate the interaction of two large-scale thermals separated along the vertical (two-dimensional problem) and the horizontal (three-dimensional problem). We will also examine the formation of air flows in the atmosphere by these thermals.

1. Formulation of the Problem. We will examine the interaction of a pair of thermals. Without going into the details of the mechanism by which they develop, we assume that two equal spherical volumes of gas are formed in the atmosphere at a certain moment of time $t = 0$. The volumes have the radius R'_0 and are filled with heated gas. Let the temperature of the gas in each volume obey the law

$$T' = T'_S + \sum_{i=1}^{2} (T'_S - T_0) \exp\left[-(bR'_i/R'_0)^2\right],$$

where R'_i is the distance from the center of the corresponding thermal; T'_S is the temperature of the gas at their centers; $T_0 = T_a$ $(z = 0)$ (here and below, the subscript denotes the parameters of a standard atmosphere [11]). The gas is assumed to be stationary at the initial moment of time: $v = 0$. Its pressure everywhere is equal to the undisturbed atmospheric pressure $p' = p_a(z)$. Let $H'_1 = R'_0$ be the height of the center of one of the thermals above the surface; and L' is the distance between the vapor centers. Thus, the height of the center of the second thermal $H'_2 = H'_1 = R'_0$, in the case of separation along the horizontal, while $-H'_2 = H'_1 + L'$.

In dimensionless variables, the initial system of equations has the form

$$\frac{d\mathbf{v}}{dt} = -\frac{1}{\gamma M^2 \rho} \nabla p + \mathbf{G} + \frac{1}{\mathrm{Re}\,\rho}\left[\Delta \mathbf{v} + \frac{1}{3}\nabla(\operatorname{div}\mathbf{v})\right],$$
$$\frac{dp}{dt} = -\gamma p \operatorname{div}\mathbf{v} + \frac{\gamma}{\mathrm{Re}\,\mathrm{Pr}}\Delta T, \tag{1.1}$$
$$\frac{dT}{dt} = -(\gamma - 1) T \operatorname{div}\mathbf{v} + \frac{\gamma}{\mathrm{Re}\,\mathrm{Pr}\,\rho}\Delta T,\ p = \rho T,\ \frac{d}{dt} \equiv \frac{\partial}{\partial t} + (\mathbf{v}\nabla),$$

where v is velocity; ρ is density; p is pressure; T is temperature; and G is a unit vector in the direction of the force of gravity.

In introducing dimensionless variables, we used the following as scales: the height of the uniform atmosphere Δ; time $\sqrt{\Delta/g}$; velocity $\sqrt{\Delta g}$; density $\rho_0 = \rho_a(0)$; temperature $T_0 = T_a(0)$; and pressure $p_0 = p_a(0)$ at the surface of the earth.

Given this choice of scales, the flow (see the initial conditions and system of equations (1.1)) is described by the determining parameters:

$$R_0 = R'_0/\Delta,\ H_1 = H'_1/\Delta,\ L = L'/\Delta,\ T_S = T'_S/T_0,$$
$$\mathrm{M} = (\Delta g/\gamma R^0 T_0)^{1/2},\ \mathrm{Re} = \Delta\sqrt{\Delta g}\rho_0/\mu,\ \mathrm{Pr} = \mu c_p/\lambda,\ \gamma. \tag{1.2}$$

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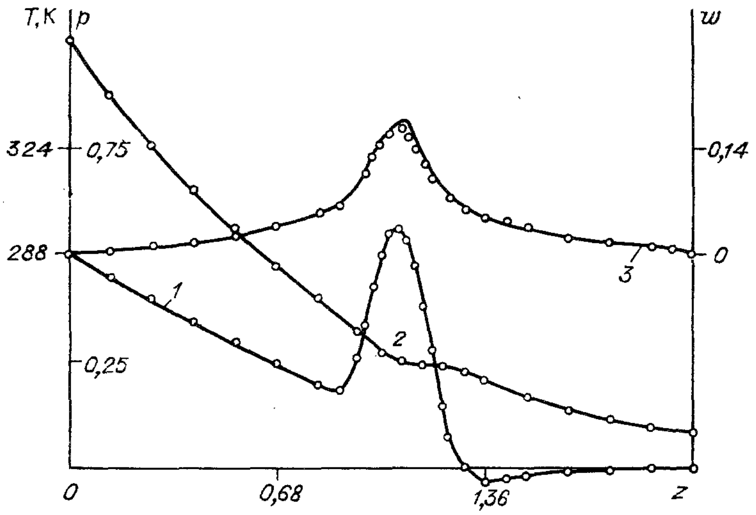

Fig. 1

Having completed the mathematical formulation of the problem, we present the boundary conditions. In the axisymmetric case (with the thermals located on one vertical and with the theoretical region being a rectangle having moving top and bottom faces; $G = \{0 \leqslant r \leqslant f_+(t),\ 0 \leqslant z \leqslant \psi_+(t)\}$) the boundary conditions are as follows:

$$\begin{aligned}
&r = f_+(t){:}\ \ u = \partial w/\partial r = 0,\ \ p = p_a(z),\ \ T = T_a(z);\\
&r = 0{:}\ \ u = \partial w/\partial r = \partial p/\partial r = \partial T/\partial r = 0;\\
&z = \psi_+(t){:}\ \ \partial u/\partial z = w = 0,\ \ p = p_a(\psi_+),\ \ T = T_a(\psi_+);\\
&z = 0{:}\ \ u = w = 0,\ \partial T/\partial z = 0.
\end{aligned} \tag{1.3}$$

In the three-dimensional case (with the thermals located on a single horizontal and the theoretical region being a rectangular parallelepiped with moving top and bottom faces (here, we assume that the vertical plane y = 0 passing through the centers of both thermals is a plane of symmetry): $V = \{f_-(t) \leqslant x \leqslant f_+(t),\ 0 \leqslant y \leqslant \varphi_+(t),\ 0 \leqslant z \leqslant \psi_+(t)\})$ the boundary conditions have the form

$$\begin{aligned}
&x = f_\pm(t){:}\ u = \partial v/\partial x = \partial w/\partial x = 0,\ p = p_a(z),\ T = T_a(z);\\
&y = 0{:}\ \ v = \partial u/\partial y = \partial w/\partial y = \partial p/\partial y = \partial T/\partial y = 0;\\
&y = \varphi_-(t){:}\ \ \partial u/\partial y = v = \partial w/\partial y = 0,\ \ p = p_a(z),\ \ T = T_a(z);\\
&z = \psi_+(t){:}\ \ \partial u/\partial z = \partial v/\partial z = w = 0,\ \ p = p_a(\psi_+),\ \ T = T_a(\psi_+);\\
&z = 0{:}\ u = v = w = 0,\ \partial T/\partial z = 0.
\end{aligned} \tag{1.4}$$

2. Method of Calculation. The above problems will be solved numerically using an explicit three-step scheme involving splitting of the physical processes in the phenomenon. Discretization is performed as follows. In the first step, the initial system of differential equations (1.1), minus the dissipative terms, is approximated on an intermediate time layer by difference equations with the use of the L ax scheme. In the second step, the sought functions are found on the top time layer for the same equations by means of a "cross" scheme. The dissipative terms are calculated in the third (final) stage.

The calculations for the three-dimensional problems were performed on relatively fine meshes: from 33 × 16 × 75 (39600 nodes) to 103 × 16 × 75 (123600 nodes). The calculations were performed on a vector-conveyor complex consisting of an ES-1055M computer linked with a MAMO processor (see [12|). Use of a vector-conveyor computer (with vectorization of the algorithm and program) made it possible to speed up the calculation by a factor of ten. The accuracy of the results was checked by checking for satisfaction of the conservation laws.

The method was tested by performing calculations for a well-known axisymmetric problem concerning the rise of a large-scale thermal in the atmosphere. Here, we compared results calculated by two- and three-dimensional methods. We examined a thermal with the parameters: $R'_0 = H'_1 = 0.17\ \Delta$, $T'_S = 12.3\ T_0$, $M = 0.2$, $Re = 10^3$, $Pr = 1$, $\gamma = 1.4$. The test calculations showed that the maximum deviations of the sought functions are small (Fig. 1, where the dis-

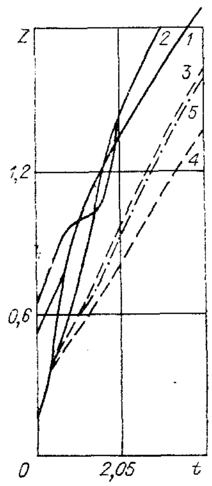

Fig. 2

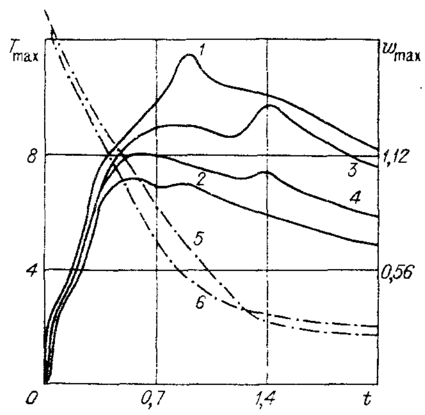

Fig. 3

tributions of temperature T(z), pressure p(z), and the vertical component of velocity w(z) (lines 1-3) on the symmetry axis of the thermal are shown for the moment of time t = 4.1 (743rd time layer); the solid curves show the three-dimensional results, while the circles represent the two-dimensional results). The values of T(z) and p(z) nearly agree, and the deviations of w(z) are no greater than 7%.

3. Rise and Interactions of a Pair of Coaxial Thermals. Let us examine the laws governing the rise and interaction of two thermals located on the same vertical axis. In this case. the determining parameters of the test problem on the rise of a single thermal are supplemented by the parameter $H_2 = H_1 + L$. The parameter L was varied within the range from $2R_0$ to $3R_0$.

Interaction between the top and bottom thermals within the given range of L is weak roughly to t = 0.34, and they rise as would their corresponding individual analogs. The nascent vortex structures then begin to interact; the evolution of the lower thermal at this stage alters the motion asymptote, the thermal ceasing to rise in accordance with the law $Z \sim t^{1/2}$ (as a single object) and beginning to move upward in accordance with $Z \sim t^2$. Here, it rapidly overtakes the upper thermal (this change in the asymptote was first noted in [13]). This development is evident in Fig. 2, where curves 1 and 2 represent the height of ascent of the pair of thermals as a function of time and correspond to $L = 2R_0$ and $3R_0$. For $L = 2R_0$, the lower thermal overtakes the upper thermal at $t \approx 0.72$. Meanwhile, since the vortex rings have not yet fully formed by this moment, the thermals appear to merge into a single structure in the interaction.

The interaction proceeds differently for $L = 3R_0$. By the moment t = 0.72, both thermals have been transformed into vortex rings with several distorted elliptical sections. The mutual effect of the thermals on one another increases greatly after this transformation, leading to significant deceleration of the upper thermal and acceleration of the bottom ring on the time interval from 0.72 to 1.45 (curve 2 in Fig. 2). The radius of the upper vortex ring gradually increases as a result of the interaction and its cross section assumes a circular form. Here, the radius of the lower ring decreases. Being greatly extended along the symmetry axis, this thermal represents a vortex structure with two cores in which vorticity has the same direction. Over time, the lower structure becomes more and more drawn out inside the top ring, this being accompanied by an increase in vorticity in its upper core and a decrease in same in the lower core; the lower core finally degenerates by $t \approx 1$. At t = 1.43, the lower ring rapidly moves through the interior opening in the upper ring and they change places. This marks the realization of the first phase of the vortex "game" (curve 2 in Fig. 2). Now, the interaction of the still-forming velocity fields results in an increase in the radius of the new upper ring and its deceleration. Conversely, the radius of the new lower ring decreases and it is accelerated. However, in contrast to the case of an ideal fluid [14]), the game ultimately ends due to the effects of diffusion of vorticity and the viscosity associated with secondary motion of the pair of rings in the compressible medium. The rings eventually merge into a single vortex (curve 2 in Fig. 2 for $t \geq 2$). This result was confirmed by experimental data obtained in [15] in a study of natural convection.

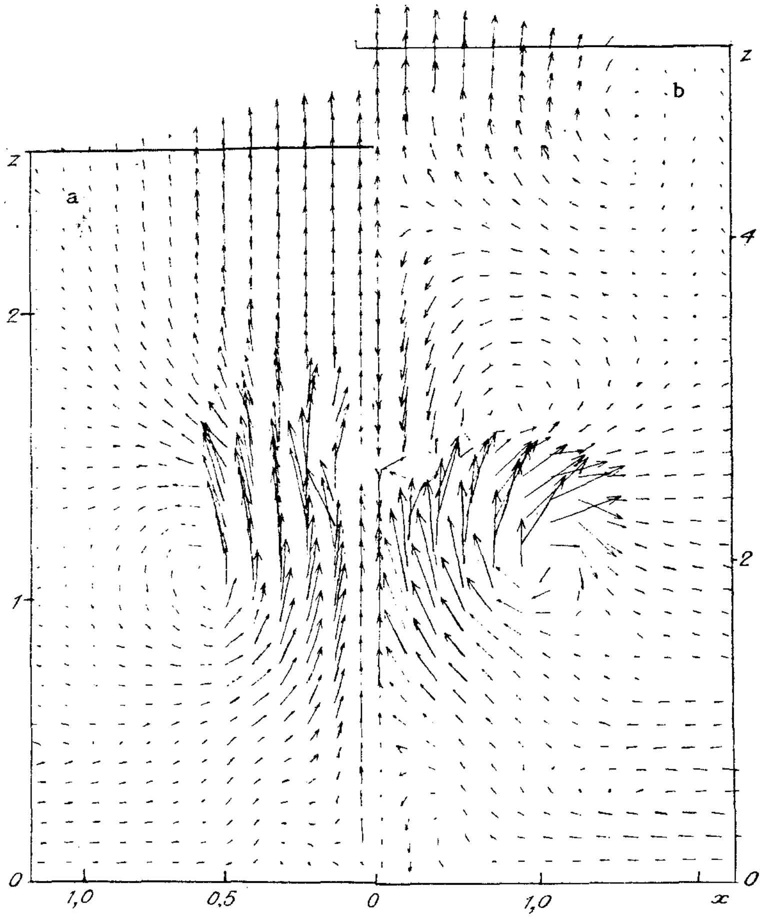

Fig. 4

Let us see how the initial distance L between the thermals in the pair affects the maximum vertical component of velocity (for the rate of rise). Figure 3 shows the dependence of w_{max} on time t (line is for the lower thermal in the pair, line 2 is for the upper thermal with $L = 2.5R_0$, and lines 3 and 4 are for the lower and upper thermals with $L = 3R_0$, respectively). It can be seen how the velocities of the lower vortex rings rapidly increase [in the form of the jump in $w_{max}(t)$] at the moment of passage through the upper rings, this phenomenon being more pronounced at smaller values of L (curves 1 and 3).

Figure 3 also shows the dependences of the maximum temperatures T_{max} in the lower and upper (lines 5 and 6) thermals on time t at $L = 2.5R_0$. The value of T_{max} initially decreases somewhat more rapidly in the upper thermal, since it is rising in a cold undistrubed medium and the lower thermal is ascending in its wake. After movement of the lower thermal through the upper thermal, they change places and temperature begins to drop more rapidly in the new upper thermal (curves 5 and 6).

An analysis of the calculated results permits the following conclusions. The ascent of a system of two large-scale coaxial thermals in the atmosphere is initiated by intensive vortical flows. A circulating flow with two cores is formed during the initial stage of ascent of the pair of thermals. Later, as a result of merging of the vortex structures, this flow is transformed into a circulating flow with a single vortex core. The course of the interaction of the thermals depends on the initial distance between them. At $L < 2.5R_0$, the resulting pair of vortex structures does not have the form of rings (the interior openings are absent), and the two merge in a manner similar to liquid drops when the lower thermal overtakes the upper thermal. A pair of annular vortices is formed at $L \geq 2.5R_0$ and a vortex "game" takes place within a certain time interval. Then in both cases the resulting monovortex rises in complete accordance with the law governing the ascent of individual structures: first in the self-similar regime ($Z \to t^{1/2}$), then slowing and fluctuating about an equilibrium position with a decaying amplitude as it generates internal waves in the atmosphere [1, 4, 5]. Here, during the concluding stage, multiple inversions of the temperature field take place. In this instance, regions with temperatures that are higher and lower than the surrounding air alternate on the symmetry axis. The oscillations of the

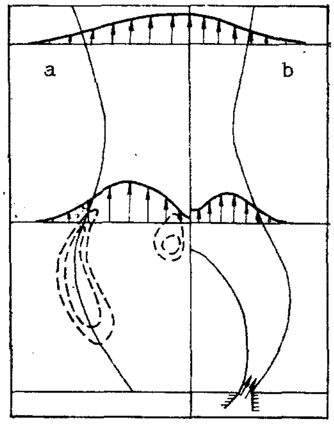

Fig. 5

cloud in the atmosphere generate a system of long, oppositely directed vortices and ascending and descending air flows. These processes continue until diffusion is completed [4, 5].

4. Rise and Interaction of Two Thermals Separated along the Horizontal. Now let us see how a pair of identical thermals located on a single horizontal axis rise and interact. In this case, the determining parameters of the problem are supplemented by the parameter $L = L'/\Delta$ (L' is the distance between the centers of the thermals; L varies within the range from $2R_0$ to $3R_0$).

As for the pair of thermals located along the vertical, L plays an important role with horizontal separation. When $L = 2R_0$, a pair of severely deformed contacting vortex rings is formed by the moment of time $t = 0.34$ (the term "vortex ring" is used quite loosely here, in view of the very small size of the interior hole). Here, the contacting sections of the rings are considerably smaller than the more distant sections and are quite a bit higher than the latter. Each vortex ring that is formed is accompanied by axial (perpendicular to the mid-section) flow through the narrow interior hole in the form of a high-intensity jet. These divergent jets begin to interact and approach one another (as occurs for pairs of divergent and parallel plane jets from nozzles [16, 17] and sprays [18]). After a certain period of time (at $t \approx 1.36$), the jets merge to form a single long high-intensity jet. However, the circulatory motion remains in this case (Fig. 4a, which shows the vector field of velocity in the left half of the plane of symmetry xOz (the vertical plane passing through the centers of both thermals) at $t = 3.1$).

Figure 5a shows the distributions of the vertical component of velocity w over x for two sections along z in the plane of symmetry at $t = 4.1$. The solid line shows the contour bounding the region where $w = 0.3\ w_{max}$. The dashed lines represent vorticity isolines. Figure 5b shows the corresponding distributions of the axial component of the velocity of the combined jet formed by the merging of two divergent jets leaving nozzles. The solid contour delimits the region with $w \approx 0.3\ w_{max}$. Shown below is the direction of the initial flow in one of the nozzles (this figure corresponds to Fig. 1 in [16]). Comparison of the images in Fig. 5a and Fig. 5b illustrates that the structures of the combined jets are completely analogous — despite the difference between the processes leading to their formation and the enormous difference in scales. Such a flow structure is maintained until attainment of the maximum altitude, where the cloud undergoes supercooling and a secondary vortex rotating in the opposite direction is formed above it (Fig. 4b, which shows the velocity field in the right side of the plane of symmetry at $t = 6.1$ in a scale different from that used in Fig. 4a). It is interesting to note that "residues" of the jet flow remain above the secondary vortex.

Now let us examine the interaction of a pair of symmetrical thermals separated along the horizontal by the distance $L = 3R_0$ at the initial moment of time. The vortex fields formed near each thermal do not interact with one another until the moment $t = 0.35$. During this time, they ascend and cool as would the corresponding individual analogs. In contrast to the variant with $L = 2R_0$, vortex rings with curved elliptical sections are partially formed before the beginning of the interaction. The closest and most distant sections of the rings in the pair are identical, i.e., they have the same dimensions and no distortion.

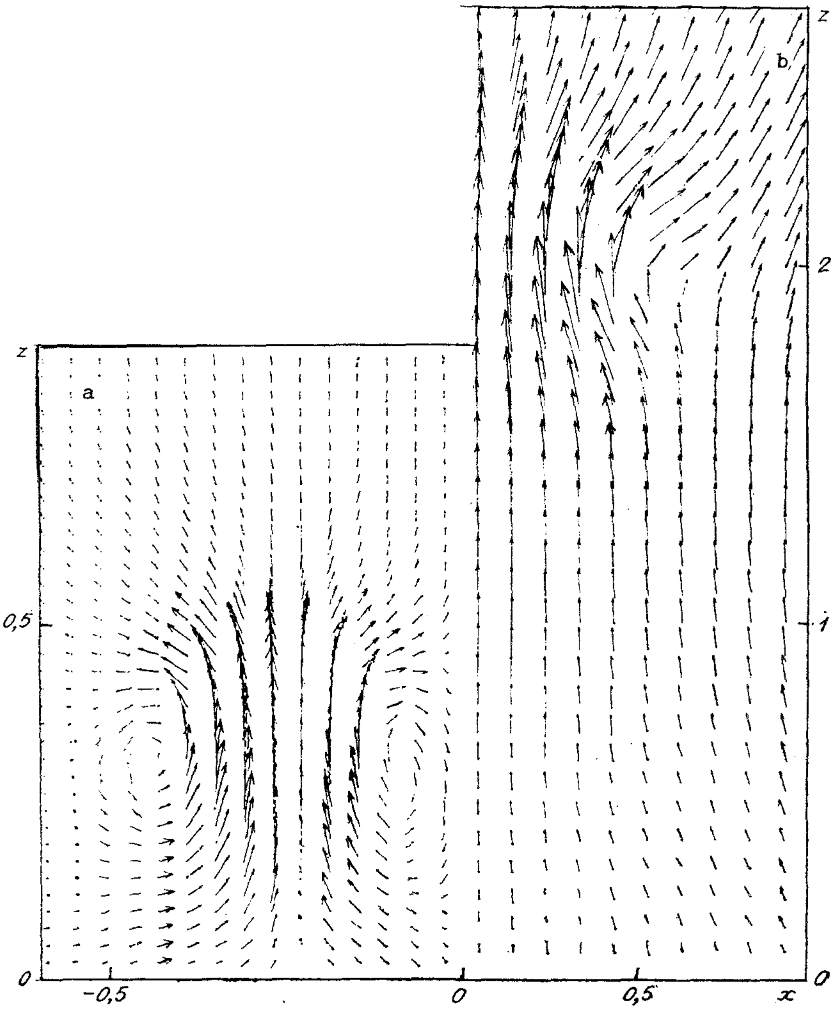

Fig. 6

Continual reinforcement of the circulating motion leads to an increase in the intensities of the jets — the central axial flows of the rings. The parallel jets are drawn to one another as a result of mutual ejection, so that the axes of the jets are inclined toward one another in the "upper" flow (in the regions where the jets leave the interior openings of the vortex rings) and merge to form a single jet in the "lower" flow. Inclination of the central jets toward one another in turn leads to inclination of the vortex rings toward each other. As a result, the closest parts of the rings are positioned somewhat below the parts that are farther from one another, i.e., the distortion noted in [10] takes place. This process of disturbance of the axial symmetry of each thermal occurs gradually, beginning roughly with the moment $t \approx 0.7$. Also, as a result of natural expansion (as is known, the radius of a ring at the self-similar stage increases in accordance with the law $R \sim t^{1/2}$), the lower parts of the vortex rings approach one another until they come into contact.

The two vortex structures subsequently merge into a single vortex with simultaneously intensifying (as a result of collision) jets that were already inclined toward one another (this process is analogous to the collision of convergent jets leaving two nozzles [19]). By the moment $t \approx 3.1$, the adjacent internal sections of the rings have disintegrated and a single vortex has formed. The formation of the single vortex owes both to reclosure of the vortex lines (see [20] for more detail) and to the entrainment of gas under the adjacent parts of the rings into the overall jet flow. The gas is drawn into the main flow by the development of low-pressure zones overhead (in the region between the interacting jets) [16-19]. Not all of the fragments of the resulting large, elongated vortex about the same plane (it is essentially a three-dimensional object): the fragments of the vortex located at the sites where the ring sections merge remain lower than the other fragments (see Fig. 5 in [8]) and two jets still flow from each of its halves.

By the moment of time $t \approx 5$, intensified entrainment of the gas under the vortex by the jet results in complete disintegration of the circulating flow and the formation of a broad divergent jet. Figure 6 shows distributions of the vector fields of velocity in the plane of symmetry xOz for two moments of time. Here, a corresponds to the moment $t = 0.7$, when two intensive vortices have already formed and have begun to interact (the left half of the field is shown). Region b in the figure corresponds to the moment $t = 5.1$, when the flow has already been completely transformed into a divergent jet (the figure shows the right half of the vector field in a scale different than that used in Fig. 6a).

Let us see how the rise of a pair of large-scale thermals in the atmosphere is influenced by their initial location in space. In [21] a simplified model was used to study the ascent of clouds from multiple nuclear explosions occurring close to one another almost simultaneously in an exponential atmosphere. The results obtained with this model indicate that the greatest altitude reached by several clouds is roughly the same as the altitude reached by a single cloud with an equivalent total energy. Our calculations show that the situation is actually considerably more complex. For example, curve 5 in Fig. 2 corresponds to the height reached by a single thermal with doubled energy as a function of time: $Z = Z(t)$. It is evident that only line 3, corresponding to the ascent of two contacting ($L = 2R_0$) thermals separated along the horizontal, is fairly close to line 5. Curve 4, corresponding to an initial separation along the horizontal $L = 3R_0$, is considerably lower than these lines. Meanwhile, the difference in altitudes increases with time. This is probably attributable to the qualitative difference in the resulting air flows. At $L < 2.5R_0$, the pair of thermals rises more rapidly and is able to reach an equilibrium position before disintegration of the circulating flow, i.e., in this sense the pair behaves as a single thermal (see Fig. 4). At $L > 2.5R_0$ (we should note that the results calculated for $L = 5R_0$, not shown here, were qualitatively similar to the results for $L = 3R_0$), the circulating motion ceases at a certain stage of evolution of the flow and an ascending expanded jet is formed (Fig. 6).

Curves 1 and 2 in Fig. 2, corresponding to pairs of coaxial thermals with $L = 2R_0$ and $L = 3R_0$, are considerably higher than curve 5. This occurs because, with vertical separation, the ascent of the flows which takes place due to the vortex "game" occurs at a substantially greater velocity.

In conclusion, we note that a check of the satisfaction of the conservation laws (for the three-dimensional problems) showed deviations of 2.2% for mass and 5.7% for energy.

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FIELD OF LONG INTERNAL LEE WAVES IN A PLANE-PARALLEL SHEAR LAYER

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A linear formulation is used to examine a three-dimensional problem on long steady-state internal waves formed by the movement of a plane-parallel shear flow over a short (relative to the depth of the water) isolated seamount. In contrast to [1, 2] (where a general linear formulation was used in a study of a field of internal lee waves in a uniform flow of an exponentially stratified fluid) and [3, 4] (where an asymptotic analysis was made of forced waves in a stably stratified flow with a velocity shift), in the present study we use a quasistatic approximation to obtain a series of double integrals representing an exact solution to the given problem for shear flow and arbitrary stable stratification of the fluid. The solution is obtained in elementary functions for a mountain of model form. Examples are presented of calculation of the near region of a field of internal lee waves in uniform and shear flows for an empirical Weisshall–Brent frequency profile.

1. Let a flow of an ideal, incompressible, stably stratified fluid of constant depth H travel from infinity with the velocity $U(z)$ to an isolated underwater obstacle $z = -H + hf(x, y)$. Meanwhile, $\max|f| = 1$, $h \ll H$, $f \to 0$ for $x^2 + y^2 \to \infty$; x and y are the horizontal coordinates; z is the vertical coordinate. The x axis is directed along the incoming flow, while the z axis is directed vertically upward. The origin of the coordinate system coincides with the undisturbed free surface.

In a quasistatic approximation, the steady-state wave field created by the obstacle in the the flow is described by the equations

$$\begin{gathered} Uu_x + wU_z = -\rho_0^{-1}p_x,\ Uv_x = -\rho_0^{-1}p_y, \\ p_z = -\rho g,\ U\rho_x + w\rho_{0z} = 0,\ u_x + v_y + w_z = 0 \end{gathered} \tag{1.1}$$

with the boundary conditions

$$p = \rho_0 g\zeta,\ U\zeta_x = w\ (z = 0),\ w = hUf_x\ (z = -H), \tag{1.2}$$

where u, v, and w are components of the vector of the wave velocities; p and ρ are perturbations of pressure and density; ζ is the displacement of the free surface; and $\rho_0(z)$ is the undisturbed density profile. The subscripts denote differentiation with respect to the corresponding coordinate. Along with (1.2), we need to satisfy the radiation condition. The latter consists of the fact that all of the principal wave disturbances are concentrated downstream ($x > 0$).

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